ENERGY BALANCE IN A PLASMA CLUSTER WHEN SCATTERED IN A VACUUM

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The plane scattering of a target acted upon by an external source of energy is considered analytically. The energy balance in the cluster during the scattering is obtained ignoring the loss due to radiation.

The scattering of a plasma cluster in a vacuum when acted upon by an external flow of energy (a laser [1], electron beams [2], and relativistic electron beams [3-5]) has been investigated in many areas of modern experimental high-energy physics. Since experiments have been carried out on the dynamics of the scattering of a target a theoretical consideration of this process is now urgent. Many important problems in this area have been investigated numerically but it is also of interest to have rougher models which can be subjected to an analytical investigation. In this paper, unlike [6, 7], we consider the steady-state scattering of a plasma cluster analytically, and we use the "mean ion" approximation to calculate the thermodynamic functions.

Model of the Interaction of an External Energy Source with a Plasma. It is convenient to distinguish three energy subsystems in a plasma cluster: free electrons, ions, and excited states of the ions (see [8]). Thermodynamic equilibrium inside these subsystems is established much more rapidly than heat exchange between the subsystems. Energy is evolved from the plasma due to different forms of radiation and is also converted into kinetic motion – scattering of the plasma cluster. A sketch showing the energy conversion in the plasma of a target is given in Fig. 1. Below we will consider only scattering of a cluster without loss due to radiation.

When the cluster is heated by a laser the energy of the external source is contributed to the free electrons. We will neglect the possibility of a non-Maxwellian "tail" of fast electrons occurring and we will assume that the imbedded energy is rapidly thermalized and is then redistributed between the subsystems.

When an electron beam or a relativistic electron beam interacts with the plasma the energy of the beam is transferred mainly to the bound electrons, and then via recombination and deexcitation of the levels to the free electrons. We will take into account the value of the energy contribution by means of a parameter specified from additional considerations. We will obtain some estimates by directing our attention to relativistic electron beams because of the large range of experimental work which has been carried out in this region.

It can be assumed that slowing down of the fast electrons occurs with approximately the same cross-section for free and bound electrons. Then, the braking ability of a beam electron $d\varepsilon/dx$ is proportional to the total electron density ZN

$$-\frac{dE}{dx} \simeq ZN \langle \sigma E \rangle . \tag{1}$$

Here E is the energy of a beam electron; Z, charge on the nucleus of the target atoms; N, density of the target nuclei; and σ, cross-section representing the energy loss; the quantity $\langle \sigma E \rangle$ should be averaged over the electron trajectory (this problem has not yet been solved).* We will start from the specified value of the energy contribution per unit volume of the target W (erg/cm^3·sec), in terms of which it is easy to express the energy flux density contributed to the target $q = Wx_0$, where x_0 is the initial thickness of the target. It follows from (1) that

*Experimental data clearly indicate that in high-current beams the depth of penetration into the target is considerably less than the path of an individual electron. This effect is obviously due to Larmor rotation of the electrons in the magnetic field induced by the beam current [3, 4].

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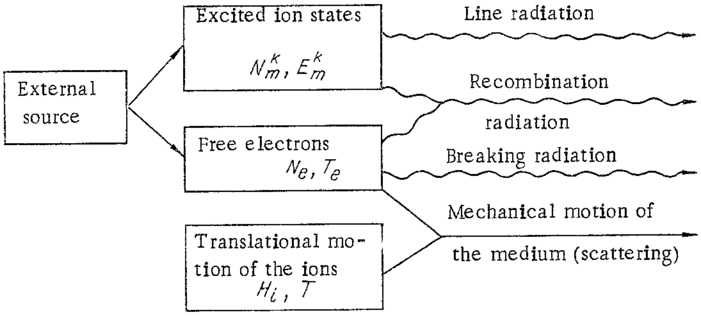

Fig. 1. Sketch of the conversion of the energy of an external source in a plasma cluster.

$$q = j_e \frac{dE}{dx} x_0 \simeq jNx \frac{1}{e} \langle \sigma E \rangle . \tag{2}$$

Here j is the beam current density. For a rectilinear electron trajectory we can use the Bethe formula for the braking ability

$$\langle \sigma E \rangle_{\text{rect}} = \frac{2\pi e^4}{(mc^2)^2} \ln \frac{\gamma^3}{2} \left(\frac{mc^2}{I} \right)^2,$$

where $\gamma = (1 - v^2/c^2)^{-1} \gg 1$; and I is the ionization potential of the principal ions of the target. Starting from the specified value of q, we can define a certain effective flux density j_{eff} such that

$$q \simeq j_{\text{eff}} Nx \frac{1}{e} \langle \sigma E \rangle_{\text{rect}}. \tag{2a}$$

Neglecting the logarithmic dependence on γ and I ($\gamma \approx 4$ for $\varepsilon \sim 1$ MeV and $I \approx 50$ eV) we provisionally obtain $j_{eff} \approx 2 \cdot 10^{10}\ q/ZNx$. As an example for $q \approx 10^{12}$ W/cm^2 (10^{19} erg/cm^2·sec), $N \approx 5 \cdot 10^{22}$ cm^{-3}, $x_0 \approx 5 \cdot 10^{-4}$ cm, and $Z \approx 80$ [2-4], we obtain $j_{eff} \approx 10^8$ A/cm^2. Note that in this case

$$\frac{j_{\text{eff}}}{j} = \frac{\langle \sigma E \rangle}{\langle \sigma E \rangle_{\text{rect}}} \sim 10^2.$$

Only this value of j_{eff} can ensure the specified energy contribution to the target (i.e., absorption of the flux q in the foil thickness).

Thermodynamic Functions of the Plasma. To describe the hydrodynamic motion of a plasma it is necessary to obtain the relationship between its macroscopic parameters such as the electron temperature T_e, ion temperature T, electron density N_e, internal energy per unit volume ε_{in}, and pressure P. We will start by considering the ionization state. We will determine the density of the most represented ions. We will assume that in view of the high density of the solid target local thermodynamic equilibrium occurs for the most represented ions, i.e., the Saha distribution for the ion multiplicities and the Boltzmann distribution for the excited states of the ions. This assumption is natural when the target is heated by a laser. When using this assumption when the target is heated with a relativistic electron beam it is necessary for the following conditions to be satisfied:

a) the presence of ionization by a beam of fast electrons should not displace the ionization equilibrium of the main ions, i.e.,

$$\nu_k = \frac{1}{e} j_{\text{eff}} \sigma_k^{\text{ion}}(E_0) \ll S_k N_e, \tag{3}$$

where ν_k (sec^{-1}) is the rate of ionization of ion k by beam electrons; σ_k^{ion}, ionization cross-section; E_0, energy of the fast electrons; $S_k(E_0)$, rate constant of plasma ionization by Maxwellian electrons; and $k \sim k_m$, charge of the main plasma ions;

b) the equilibrium with respect to ionization multiplicities should not be disturbed due to scattering

$$S_k N_e \frac{x}{v} \gg 1,\ k \sim k_m; \tag{3a}$$

c) radiational transitions should have no effect on the Boltzmann distribution of the populations of the excited states of the most represented ions, i.e.,

$$N_e \gg A_n^k/V_n^k,\ k \sim k_{\mathrm{m}} \tag{3b}$$

for all the levels n; A_n^k (sec^{-1}) and V_n^k (cm$^3\cdot$sec^{-1}) are the probability and rate constant of the radiational and collisional decay of the level n, respectively.

To calculate the thermodynamic functions of the plasma we will assume that conditions (3) are satisfied, and we will demonstrate their correctness later by means of specific estimates.

The Saha distribution of the ions with respect to the multiplicities has the form*

$$\frac{N_e N_{k-1}}{N_k} = \frac{S_{k-1}}{\beta_k} = 6\cdot 10^{21} T_e^{3/2} \exp(-I_k/T_e), \tag{4}$$

where β_k (cm$^6\cdot$sec^{-1}) is the rate constant of triple recombination of the ions on electrons. Further, for convenience, we will approximate the ionization energy of the ion K by the formula $I_k = A(Z, k)\, k^2$, where $A(Z, k)$, eV is an adjusting constant (Z is the charge of the nucleus of the ions). With this multiplicity the most represented ion (where $N_k \sim N_{k-1}$, $N_e \sim k_m N$) can be estimated from the equation

$$k_{\mathrm{m}} \approx \left[\frac{T_e}{A} \ln\left(\frac{6\cdot 10^{21} T_e^{3/2}}{k_{\mathrm{m}} N}\right)\right]^{1/2} \approx \sqrt{\frac{8}{A}}\, T_e^{1/2}. \tag{5}$$

This expression holds approximately in the region $10 < T_e < 200$ eV and $10^{20} \le N_e \le 10^{22}$ cm^{-3}. Bearing in mind relation (3) and Pitaev's equation [9] for the rate of triple recombination of an electron, we obtain the relaxation rates for the ions $k \sim k_m$

$$\beta_k \sim \frac{S_k}{N_e} \sim 10^{-27} k_{\mathrm{m}}^3/T_e^{9/2} \simeq \left(\frac{8}{A}\right)\cdot 10^{-27} T_e^{-3}. \tag{6}$$

Considering the plasma scattering foil as an ideal medium† characterized by a "mean charge" $k \sim k_m$, we can write approximate equations for the thermodynamic functions of the plasma. The thermal energy E_T of unit volume

$$E_T = \frac{3}{2}(N_e T_e + NT) \approx 3\sqrt{\frac{2}{A}}\, N T_e^{3/2}. \tag{6a}$$

We will calculate the ionization energy of the plasma

$$E_{\mathrm{ion}} = \Big(\sum_{k=0}^{Z} N_k \sum_{k'=0}^{k} A k'^2\Big) \approx \frac{16}{3}\sqrt{\frac{2}{A}}\, N T_e^{3/2}, \tag{6b}$$

in which case the total internal energy is

$$E_{\mathrm{in}} = E_T + E_{\mathrm{ion}} \simeq \frac{25}{3}\sqrt{\frac{2}{A}}\, N T_e^{3/2} \approx 1.6\cdot 10^{-12}\, \frac{25}{3}\sqrt{\frac{2}{A}}\, N T_e^{3/2}\ [\mathrm{erg/cm^3}]$$

and

$$T_e\,[\mathrm{eV}] = 1.78\cdot 10^7 \left(\frac{A}{2}\right)^{1/3} (E_{\mathrm{in}}/N)^{2/3}. \tag{6c}$$

The pressure of the plasma

$$P = (N_e T_e + NT) \sim 2\sqrt{\frac{2}{A}}\, N T_e^{3/2}. \tag{6d}$$

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* In practical formulas the energy characteristics of the ions and electrons, and also the temperatures, are measured in eV, while the remaining quantities (where this is not stated) are in cgs units.

† A plasma of solid-state density with temperature $T_c \leqslant 3$ eV, as estimates using the relation $N_{eid} \ll 3\cdot 10^{20}\cdot (T_e/k_m)^3$ show, is nonideal. However, in a time $t_h \sim 10^{-8}$ sec the plasma is heated (see the paragraph entitled "Model of plasma scattering") up to $T_e \gtrsim 3$ eV and is scattered to a density $N_e \sim 10^{22}$ cm^{-3}, at which it can be regarded as an ideal gas ($N_{eid} \sim 10^{22}$ cm^{-3}). The main energy of the external source in a time $\tau \gg t_h$ is introduced therefore into an ideal plasma.

In the estimates (6) we took into account the fact that $N_e \sim k_m N$, $k_m T_e \geq T$. In these approximations the heat capacity of the plasma depends on the temperature

$$c_v = \left(\frac{\partial E_{\text{in}}}{\partial T_e}\right)_v = \frac{25}{2}\sqrt{\frac{2}{A}}NT_e^{1/2}. \tag{7}$$

Equations (3) and (6) are of course only satisfied if the plasma ions are not completely stripped, i.e., $k_m < Z$.

We will now consider the interesting practical case of small Z when the most representative ions are completely ionized almost from the beginning of the action of the external pulse. Then $N_e \simeq ZN$ and

$$E_T \simeq \frac{3}{2}(ZT_e + T)N \sim \frac{3}{2}ZNT_e, \; E_{\text{ion}} \approx N\Delta I, \tag{8}$$

$$E_{\text{in}} \sim N\Delta I + \frac{3}{2}NZT_e, \tag{8a}$$

where ΔI is the total ionization energy of atom Z, and

$$P \approx ZNT_e. \tag{8b}$$

It can be seen that in this case the temperature dependence of the heat capacity of the plasma disappears.

It follows from these results that the internal energy and the pressure are proportional $P \simeq (6/25)E_{\text{in}}$. This effect considerably simplifies the consideration of the problem. For example, multiplying the equation of motion by the velocity $v = dx/dt$, we obtain a useful expression relating the kinetic energy $E_{\text{kin}} = M_0 v^2/2$ and the internal energy

$$\frac{1}{E_{\text{in}}}\frac{dE_{\text{in}}}{dt} = -\frac{6}{25}\frac{1}{x}\frac{dx}{dt}. \tag{9}$$

In order to show the effect of radiation on the dynamics of the scattering we will consider two limiting cases: scattering without radiation losses and scattering with volume losses without reabsorption.

Plasma Scattering Model. 1. Initial Equations. Consider plane scattering of a target subject to the action of a volume source of energy dissipation. The volume nature of the energy dissipation in the case of a relativistic electron beam is ensured both due to the path of the electrons in the depth of the target, and due to the electron thermal conductivity, while in the case of a laser it is solely due to thermal conductivity, as has already been discussed in the literature [1]. Assuming that the target is fairly thin we will assume that all the physical quantities are uniform over the thickness of the target. In this approximation scattering of the cluster will be described by the following equations:

$$\mu N x = \widetilde{M}_0, \; \widetilde{M}_0\frac{dv}{dt} = P, \; \frac{d}{dt}\left(\widetilde{M}_0\frac{v^2}{2} + \widetilde{E}_{\text{in}}\right) = q - q_{\text{rad}}. \tag{10}$$

Here μ is the mass of the heavy particle; $\widetilde{M} = \mu N x$, $\widetilde{M}_0 = \mu N_0 x_0$, $\widetilde{E}_{\text{in}} = E_{\text{in}}x$, surface density of the internal energy; N_0, N, initial and current values of the density of the target material; $q_{\text{rad}} = Q_{\text{rad}}x$, flux density of the radiation from the plasma; Q_{rad}, total radiational losses from unit volume; x, coordinate; and t, time. In (10) we have also assumed that

$$\partial P/\partial x \approx P/x. \tag{11}$$

2. Scattering without Radiation. We will put $q = q_s t^s$, where q_s and s are certain constants, and $q_{\text{rad}} = 0$. System (10) takes the form

$$\widetilde{M}_0 x\ddot{x} = \frac{6}{25}\widetilde{E}_{\text{in}}, \; \frac{d}{dt}\left(\frac{\widetilde{M}_0\dot{x}^2}{2} + \widetilde{E}_{\text{in}}\right) = q_s t^s. \tag{12}$$

We will seek a steady-state solution in the form of power relations $x \sim t^\alpha$, $\widetilde{E}_{\text{in}} \sim t^\beta$. The constants α and β are chosen so that the time dependences of the left and right sides in (12) are the same. This condition is satisfied when $2\alpha - 2 = \beta$, $\beta - 1 = s$. Hence, $\beta = s + 1$, $\alpha = (s + 3)/2$. Hence, the steady-state solutions are

$$\widetilde{E}_{\text{in}} = \varepsilon_s t^{s+1}, \quad x = y_s t^{\frac{s+3}{2}}, \tag{13}$$

where ε_s and y_s are given by the equations

$$\widetilde{M}_0(s+1)(s+3)y_s^2 = \varepsilon_s\frac{24}{25}, \quad \frac{1}{8}\widetilde{M}_0(s+3)\,y_s^2 + \varepsilon_s = \frac{q_s}{s+1}, \tag{14}$$

whence we obtain

$$\varepsilon_s = 25\,\frac{q_s}{(28s+34)}, \quad y_s = [12q_s/(14s+17)(s+3)(s+1)\widetilde{M}_0]^{1/2}. \tag{15}$$

From their physical meaning the quantities q_s, ε_s, and y_s are positive. Consequently, the region of possible values of s for the steady-state solutions is limited by the condition $s > -1$.

From (13) and (9) we have the following ratio of the kinetic energy and the internal energy of the plasma in the steady-state mode:

$$\widetilde{E}_{\text{kin}}/\widetilde{E}_{\text{in}} = 3(s+3)/25(s+1). \tag{16}$$

The time dependence of the plasma temperature follows from relation (6c) $\tilde{E}_{\text{in}} \backsim T_e^{3/2}$. Hence,

$$T_e(t) = \left(\frac{3\cdot 10^{12} q_s}{(28s+34)\,1.6\sqrt{\frac{2}{A}}\,\widetilde{N}_0}\right)^{2/3} t^{\frac{2(s+1)}{3}}\ [\text{eV}]. \tag{17}$$

We will introduce the instant of time t_0 such that when $t = t_0$ the coordinate takes the initial values $x(t_0) = x_0$. Then

$$x = x_0\left(\frac{t}{t_0}\right)^{\frac{s+3}{2}}, \quad \widetilde{E}_{\text{in}}(t) = \widetilde{E}_0\left(\frac{t}{t_0}\right)^{s+1}, \quad T_e = T_{e0}\left(\frac{t}{t_0}\right)^{\frac{2(s+1)}{3}}, \tag{18}$$

where $\widetilde{E}_0 = \varepsilon_s t_0^{s+1}$, $x_0 = y_s t_0^{\frac{s+3}{2}}$, etc. We will assume the initial size of the plasma x_0 to be specified and we will express the other quantities in terms of it:

$$t_0 = \left(x_0\sqrt{\frac{\widetilde{M}_0}{q_s}}\right)^{\frac{2}{s+3}}\left(\frac{(14s+17)(s+3)(s+1)}{12}\right)^{\frac{1}{s+3}},$$

$$\widetilde{E}_0 = \frac{25q_s}{28s+34}\,t_0^{s+1},$$

$$T_{e0} = \left(\frac{3\cdot 10^{12} q_s}{(28s+34)\cdot 1.6\sqrt{\frac{2}{A}}\,\widetilde{N}_0}\right)^{2/3} t_0^{\frac{2(s+1)}{3}}.$$

The quantity t_0 represents the time taken for arbitrary initial parameters to reach the steady-state mode. We can show this by considering the relaxation of small deviations δx and $\delta\widetilde{E}_{\text{in}}$ from solutions (13). In fact, we obtain from (12)

$$\ddot{x}\delta x + x\delta\ddot{x} = \frac{6}{25}\,\frac{\delta\widetilde{E}_{\text{in}}}{\widetilde{M}_0}, \quad \widetilde{M}_0\dot{x}\delta\dot{x} + \delta\widetilde{E}_{\text{in}} = 0, \tag{20}$$

where x(t) is given by Eqs. (13), (15). Equations (20) can be converted to the form

$$\delta\ddot{x} + \frac{24}{25}\,\frac{s+3}{8}\,t^{-1}\delta\dot{x} + \frac{(s+3)(s+1)}{4}\,t^{-2}\,\delta x = 0.$$

The characteristic relaxation time is simply a quantity of the order of the scattering time t. At the initial instant $t \sim t_0$ when $x \sim x_0$, and consequently t_0 represents the relaxation time of the initial parameters to the values specified by Eqs. (19).

Consider the scattering of a target of light atoms. For a large value of the energy contribution and small Z of the target atoms the plasma, during practically the whole time during which the external source acts, consists of bare nuclei and free electrons. Using the thermodynamic functions of such a plasma (9) and ignoring the losses due to radiation, we obtain

$$\widetilde{M}_0 x\ddot{x} = \frac{2}{3}\widetilde{E}_{\text{in}}, \quad \frac{\widetilde{M}_0\dot{x}^2}{2} + \widetilde{E}_{\text{in}} = q_s\frac{t^{s+1}}{s+1} - N_0\Delta I. \tag{21}$$

Here $\widetilde{E}_{\text{in}} = (3/2)Px$, where $N_0\Delta I$ is the total ionization energy of the target atoms. The characteristic total-ionization time $t_1 \simeq \left(\frac{N_0\Delta I(s+1)}{q_s}\right)^{1/s+1}$. When $t \gg t_1$ it is easy to show that the solutions of (21) have the same form: $x = y_s t^{\frac{s+3}{2}}$, $E_{\text{in}} = \varepsilon_s t^{s+1}$. We have for y_s and ε_s

$$\varepsilon_s = \frac{3q_s}{2(2s+3)}, \quad y_s = [4q_s/(s+1)(s+3)(2s+3)\widetilde{M}_0]^{1/2}. \tag{22}$$

From (9) we obtain the following relation between E_{kin} and E_{in}:

$$E_{\text{kin}}/E_{\text{in}} = (s+3)/3(s+1). \tag{23}$$

Hence, for a constant external flux $q = q_0$ ($s = 0$), $9/(25+9)$ of the total energy contribution is converted into kinetic energy of motion of a cluster with large atomic weight, while $25/34$ is converted into internal energy of the cluster. For scattering under these conditions $3/(3+3) = 0.5$ of the total energy contribution is converted into energy of motion of a cluster of light atoms, while the remaining part of the energy goes into heating and ionizing the cluster.

The dependence of the energy contribution on time has a considerable effect on the energy distribution between the different subsystems. It can be seen from (16) that when s is reduced the fraction of the kinetic energy increases. When $s \gg 1$ the kinetic energy is $\sim 12\%$ of the total internal energy of the plasma; for $s = 0$, $E_{\text{kin}}/E_{\text{in}} \approx 27\%$; for an energy contribution that falls with time ($s < 0$), it is obviously possible to have a mode in which the plasma is accelerated with weak heating ($E_{\text{kin}}/E_{\text{in}} \gg 1$ as $s \to -1$).

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